INDUSTRIAL & OPERATIONS ENGINEERING
UNIVERSITY OF MICHIGAN

Frequency Stability-Constrained Unit Commitment: Tight Approximation using Bernstein Polynomials

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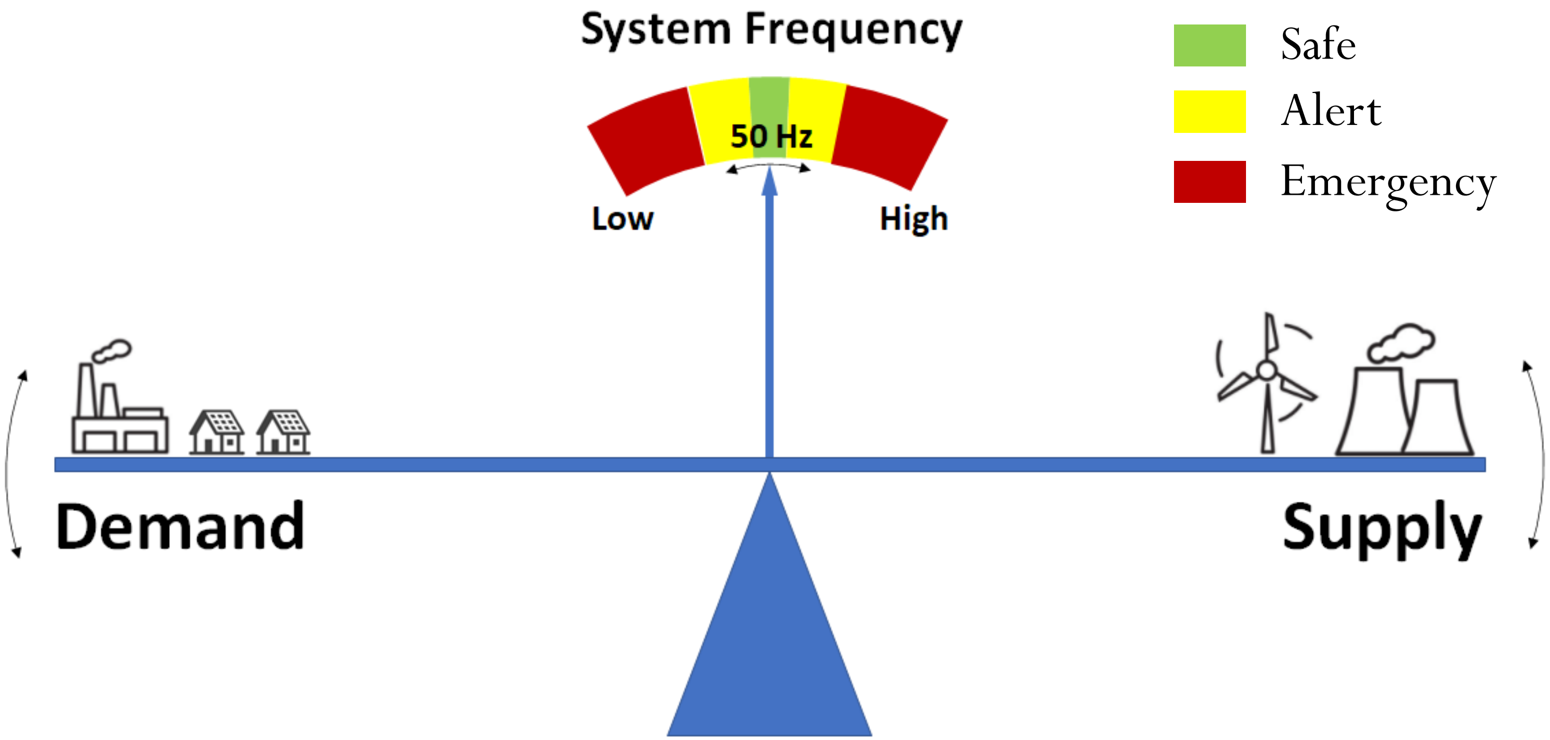
Outline

- ❑Introduction and Problem Formulation
- ❑Solution Method
- ❑Case Study
- ❑Conclusions

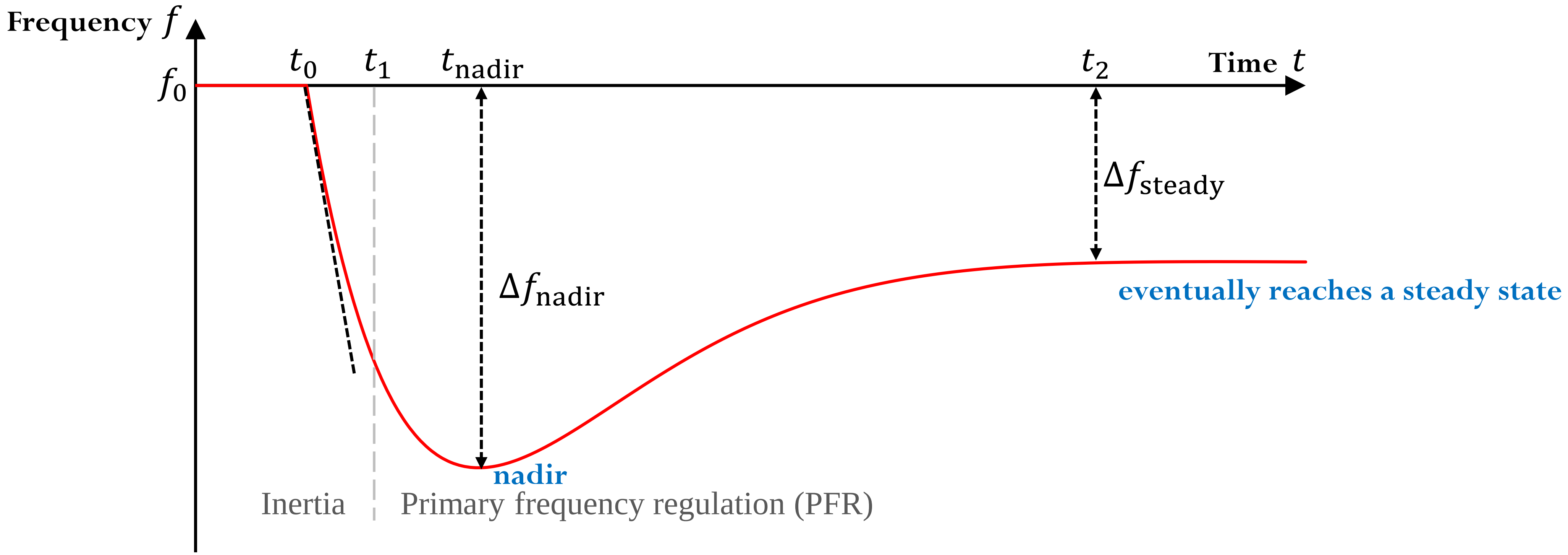
Power System Frequency



The frequency of power systems should be maintained closely around the nominal value

General Frequency Dynamics

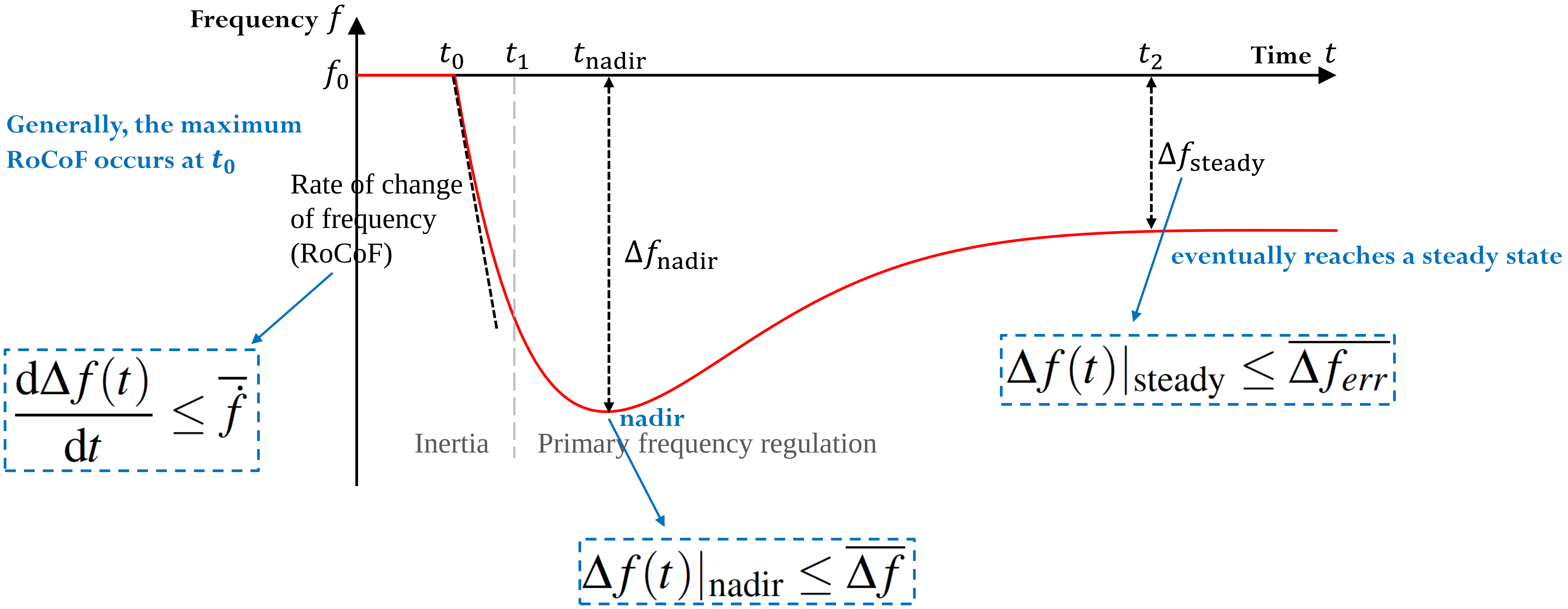
Frequency dynamics during an under-frequency event (a sudden loss of generation at t_0)



- $t_0 - t_1$: Inertia plays a major role in mitigating frequency drop (PFR does not respond effectively)
- $t_1 - t_2$: PFR becomes significant, and the frequency eventually reaches a steady state
- t_{nadir}: Time to reach the nadir during the whole dynamics

Frequency Security Metrics

Frequency dynamics during an under-frequency event (a sudden loss of generation at t_0)



Governing Equations

Governing equations of system frequency dynamics

inertia — load damping — imbalance — PFR

$$2H_{sys}\frac{d\Delta f(t)}{dt} + k_D P_d \Delta f(t) = \Delta P_d - P_{sys}^{PR}(t)$$

$$\Delta f(t)|_{t=0} = 0$$ **(Initial condition)**

Total PFR power

$$P_{sys}^{PR}(t) = \sum_i \left[\underline{P_{g,i}^{PR}(t)} + \underline{P_{w,i}^{PR}(t)} \right]$$

$$T_{g,i}\frac{dP_{g,i}^{PR}(t)}{dt} + P_{g,i}^{PR}(t) = G_{g,i} I_{g,i} \Delta f(t)$$

$$P_{g,i}^{PR}(t)|_{t=0} = 0$$ **(Initial condition)**

$$P_{w,i}^{PR}(t) = G_{w,i}\Delta f(t)$$

PFR power from thermal units — PFR power from wind farms

Notation:
- i: bus index
- k_D: load damping rate
- P_d: total power load
- ΔP_d: power imbalance
- T_g / T_w: response constant
- G_g / G_w: droop factor
- H_{sys}: total inertia
- Δf: frequency deviation
- P_{sys}^{PR}: total PFR power
- P_g^{PR} / P_w^{PR}: PFR power
- I_g: online/offline status

Frequency-Secured Unit Commitment

Objective: Minimize operation cost

$$\min \sum_{\tau}\sum_{i}\Big(\underbrace{c_{su,i}U_{g,i,\tau}+c_{sd,i}D_{g,i,\tau}}_{\text{startup \& shutdown cost}}+\underbrace{c_{g}^{PR}R_{g,i,\tau}^{PR}+c_{w}^{PR}R_{w,i,\tau}^{PR}}_{\text{PFR reserve cost}}\Big)+\sum_{\tau}\sum_{i}\sum_{s}\underbrace{\omega_{s}F_{g,i,\tau}^{s}}_{\text{expected fuel cost}}$$

Subject to:

- Piecewise linearization of fuel cost calculation
- Logic constraint of unit status
- Minimum online & offline time constraint
- **Frequency security constraint**
- Generation and ramping constraint
- Power balance constraint
- DC power flow constraint

Ref: B. Zhou, J. Fang, X. Ai, et al, "Partial-dimensional correlation-aided convex-hull uncertainty set for robust unit commitment," IEEE Transactions on Power Systems, 38(03), 2434-2446, 2023.

Frequency-Secured Unit Commitm

Notation:
s: scenario index
τ: period index
$c_{(\cdot)}$: cost coefficient
ω_s: probability of scenario s
P_w^A: available wind power
H_g/H_w: inertia constant
P_w: integrated wind power
R_g^{PR}/R_w^{PR}: PFR reserve

Objective: Minimize operation cost

$$\min \sum_{\tau}\sum_{i}\Big(\underbrace{c_{su,i}U_{g,i,\tau}+c_{sd,i}D_{g,i,\tau}}_{\text{startup \& shutdown cost}}+\underbrace{c_g^{PR}R^{PR}_{g,i,\tau}+c_w^{PR}R^{PR}_{w,i,\tau}}_{\text{PFR reserve cost}}\Big)+\sum_{\tau}\sum_{i}\sum_{s}$$

Subject to:

- Piecewise linearization of fuel cost calculation
- Logic constraint of unit status
- Minimum online & offline time constraint
- **Frequency security constraint**
- Generation and ramping constraint
- Power balance constraint
- DC power flow constraint

Governing equations
Frequency security metrics

$$H_{sys,\tau}=\sum_{i}\left(H_{g,i}I_{g,i,\tau}+H_{w,i}\right)$$

$$P^{PR}_{g,i,\tau}(t)\le R^{PR}_{g,i,\tau}\qquad P^{PR}_{w,i,\tau}(t)\le R^{PR}_{w,i,\tau}$$

$$0\le P^{s}_{w,i,\tau}\le P^{A,s}_{w,i,\tau}-R^{PR}_{w,i,\tau}$$

Ref: B. Zhou, J. Fang, X. Ai, et al, "Partial-dimensional correlation-aided convex-hull uncertainty set for robust unit commitment," IEEE Transactions on Power Systems, 38(03), 2434-2446, 2023.

DAE-Constrained Optimization

Objective: Minimize operation cost

$$\min \sum_{\tau}\sum_{i}\Big(\underbrace{c_{su,i}U_{g,i,\tau}+c_{sd,i}D_{g,i,\tau}}_{\text{startup \& shutdown cost}}+\underbrace{c_g^{PR}R_{g,i,\tau}^{PR}+c_w^{PR}R_{w,i,\tau}^{PR}}_{\text{PFR reserve cost}}\Big)+\sum_{\tau}\sum_{i}\sum_{s}\underbrace{\omega_s F_{g,i,\tau}^{s}}_{\text{expected fuel cost}}$$

Subject to: Two types of constraints

- **Discrete-time constraints** – mixed-integer linear equations, tractably handled by solvers
- **Continuous-time constraints – differential algebraic equations (DAE)**

$$2H_{sys}\frac{\mathrm{d}\Delta f(t)}{\mathrm{d}t}+k_D P_d \Delta f(t)=\Delta P_d - P_{sys}^{PR}(t)$$

$$\Delta f(t)|_{t=0}=0$$

$$T_{g,i}\frac{\mathrm{d}P_{g,i}^{PR}(t)}{\mathrm{d}t}+P_{g,i}^{PR}(t)=G_{g,i}I_{g,i}\Delta f(t)$$

$$P_{g,i}^{PR}(t)|_{t=0}=0$$

$$\frac{\mathrm{d}\Delta f(t)}{\mathrm{d}t}\le \overline{\dot{f}}$$

$$P_{w,i}^{PR}(t)=G_{w,i}\Delta f(t)$$

$$P_{g,i,\tau}^{PR}(t)\le R_{g,i,\tau}^{PR} \qquad P_{w,i,\tau}^{PR}(t)\le R_{w,i,\tau}^{PR}$$

$$\Delta f(t)|_{\text{nadir}}\le \overline{\Delta f} \qquad \Delta f(t)|_{\text{steady}}\le \overline{\Delta f_{err}}$$

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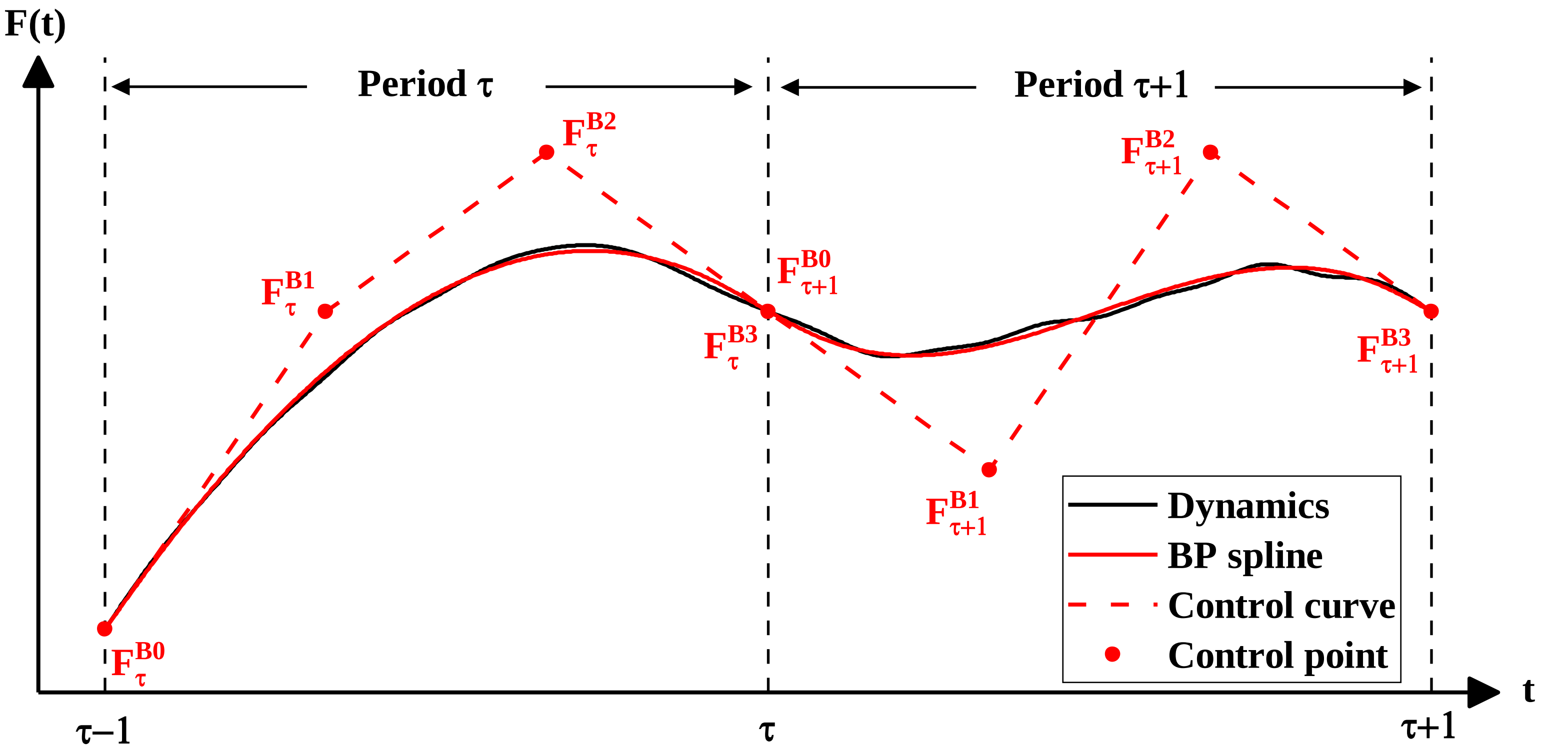
Bernstein Polynomial Approximation

Core idea: Use Bernstein polynomial (BP) spline to approximate dynamics



BP spline $F(t) = \sum_{k=0}^{3} F^{B,k} B_{3,k}(t) = (F^B)^{\mathrm{T}} B_3(t), t \in [0,1]$

Cubic BP $B_{3,k}(t) \coloneqq \binom{3}{k} t^k (1-t)^{3-k}, t \in [0,1]$

Transformation – Part 1

According to $F(t) = (F^B)^{\mathrm{T}} B_3(t)$, we have

- Integral term

  from 0 to 1

$$\int_0^1 F(t)\mathrm{d}t = (F^B)^{\mathrm{T}} \int_0^1 B_3(t)\,\mathrm{d}t = 1^{\mathrm{T}}\, F^B/4$$

- Derivative term

$$\frac{\mathrm{d}F(t)}{\mathrm{d}t} = 3[F^{B,1} - F^{B,0}, F^{B,2} - F^{B,1}, F^{B,3} - F^{B,2}]\boldsymbol{B}_2(t) = \left(\boldsymbol{W}_3\boldsymbol{F^B}\right)^{\mathrm{T}}\boldsymbol{B}_2(t)$$

quadratic BP

- Equality equation

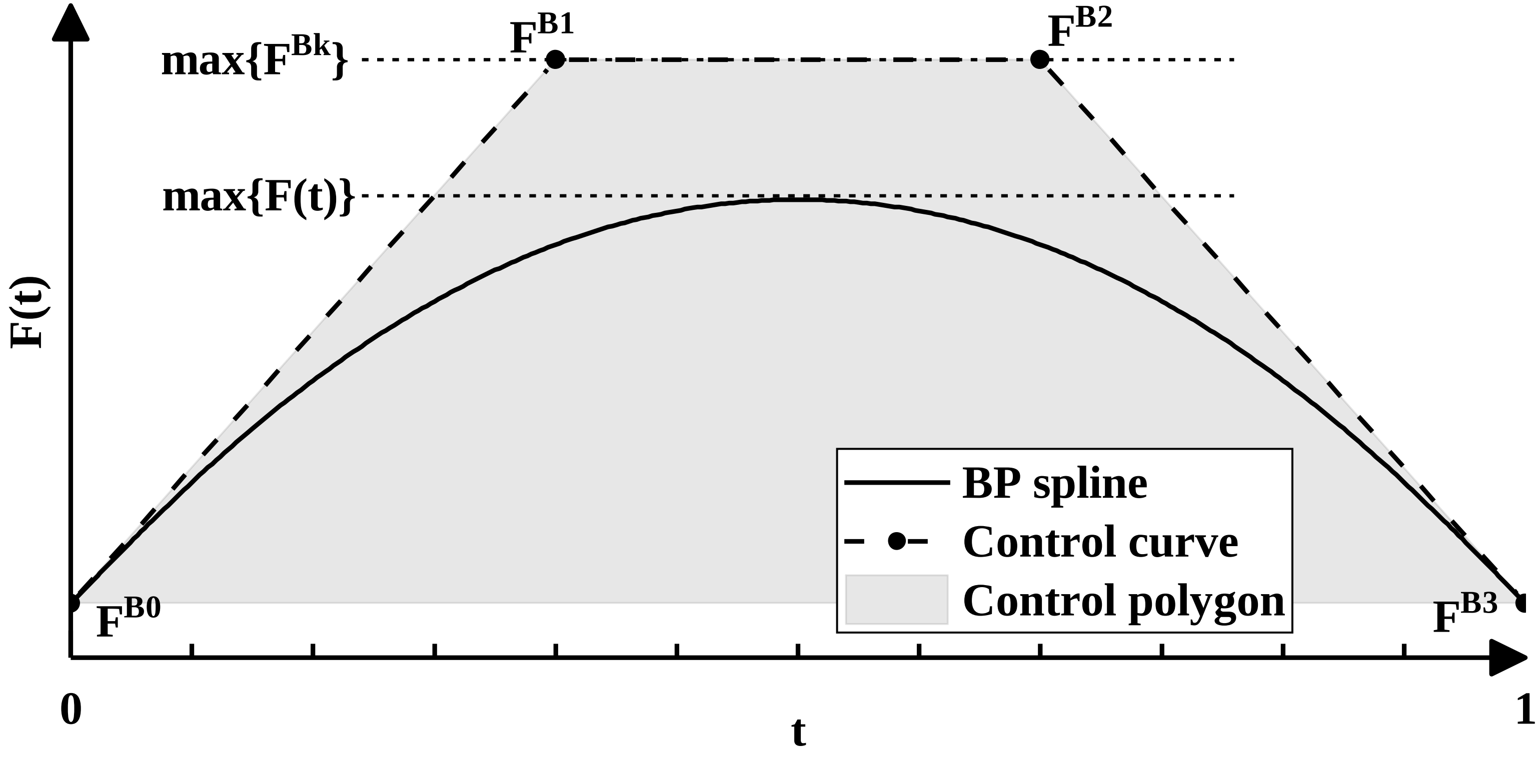
$$F(t) = 0 \Leftrightarrow \left(\boldsymbol{F^B}\right)^{\mathrm{T}}\boldsymbol{B}_3(t) = 0 \Leftrightarrow F^{B,k} = 0$$

undetermined coefficient method

How about inequality equations and ODEs?

Convex-hull Property of BP

BP splines must be inside their corresponding control polygons

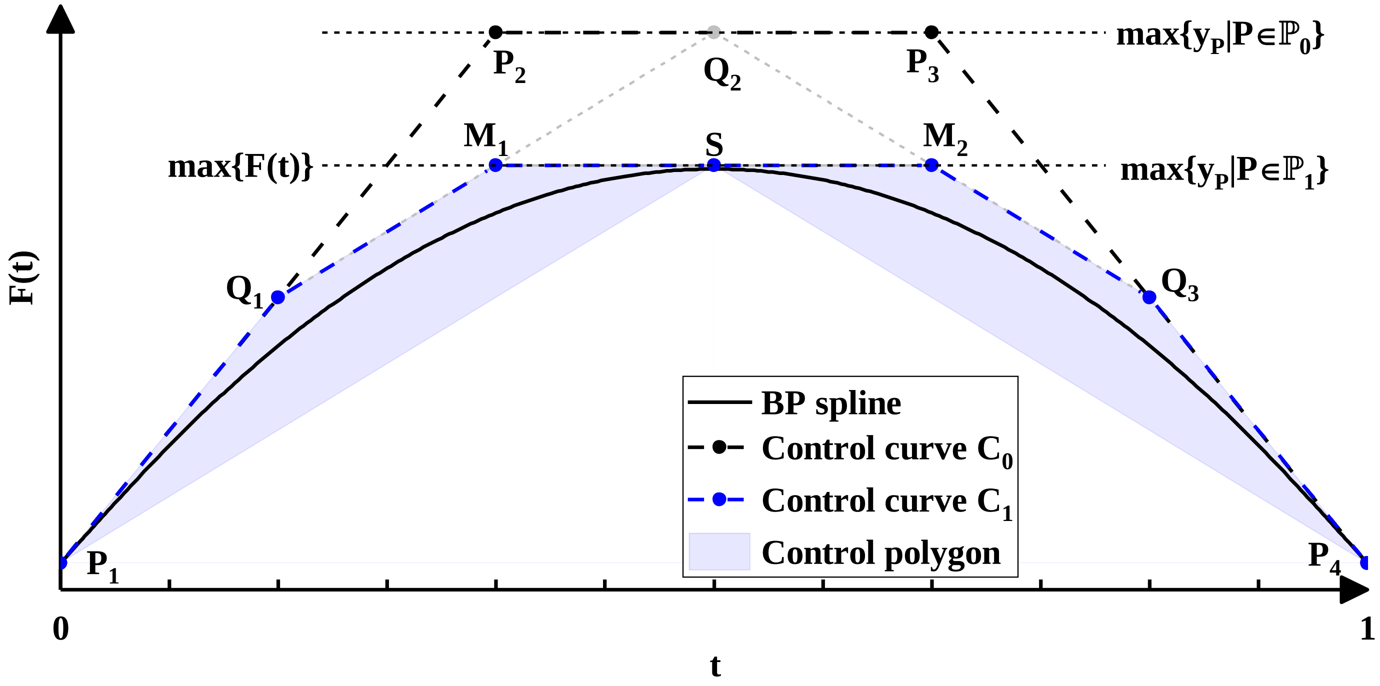


$$\min\{F^{B,k}\} \le F(t) \le \max\{F^{B,k}\}$$

➢ Inequality equation $F(t) \le c \Leftarrow \max\{F^{B,k}\} \le c \Leftrightarrow F^{B,k} \le c$

Subdivision of BP

Break BP splines into several segments, then each segment is still a BP spline



Notation:

$\mathbb{P}_i$: set of control points after i subdivision

y_P: value of control point P

$Y^{(i)}$: vector of ordinate of control points after i subdivision

de Casteljau's algorithm

$$\begin{cases} \overrightarrow{P_1Q_1} = t_0\overrightarrow{P_1P_2}, \overrightarrow{P_2Q_2} = t_0\overrightarrow{P_2P_3}, \overrightarrow{P_3Q_3} = t_0\overrightarrow{P_3P_4} \\ \overrightarrow{Q_1M_1} = t_0\overrightarrow{Q_1Q_2}, \overrightarrow{Q_2M_2} = t_0\overrightarrow{Q_2Q_3} \\ \overrightarrow{M_1S} = t_0\overrightarrow{M_1M_2} \end{cases}$$

Assume $Y^{(0)} = F^B$, we have $Y^{(1)} = AY^{(0)}$

$$\boldsymbol{A_l} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1-t_0 & t_0 & 0 & 0 \\ (1-t_0)^2 & 2\tau_0(1-t_0) & t_0^2 & 0 \\ (1-t_0)^3 & 3\tau_0(1-t_0)^2 & 3t_0^2(1-t_0) & t_0^3 \end{bmatrix}$$

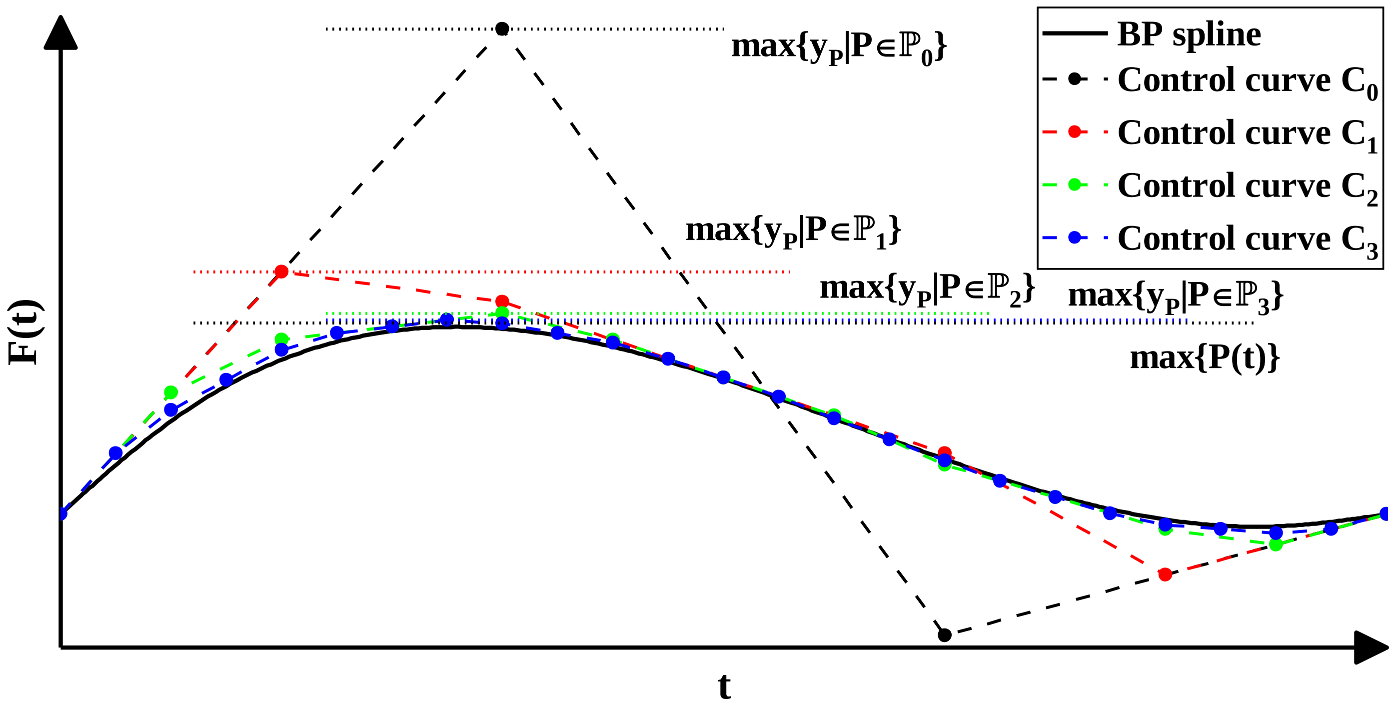
$$\boldsymbol{A_r} = \begin{bmatrix} (1-t_0)^3 & 3\tau_0(1-t_0)^2 & 3t_0^2(1-t_0) & t_0^3 \\ 0 & (1-t_0)^2 & 2t_0(1-t_0) & t_0^2 \\ 0 & 0 & 1-t_0 & t_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{A} = [\boldsymbol{A}_l^{\mathrm{T}}, \boldsymbol{A}_r^{\mathrm{T}}]^{\mathrm{T}}$$

Ref: W. Boehm and A. Mller, "On de Casteljau's algorithm," Computer Aided Geometric Design, vol. 16, no. 7, pp. 587–605, 1999.

Transformation – Part 2

By **repeatedly** implementing the subdivision property, we can narrow the gap between BP splines and control curves



$$Y^{(m)} = D^{(m)}Y^{(m-1)}, m \geq 1$$

$$D^{(m)} = \begin{bmatrix} A & & \\ & \ddots & \\ & & A \end{bmatrix} (2^{m-1} A)$$

$$Y^{(m)} = D^{(m)} \cdots D^{(1)} Y^{(0)} = E^{(m)} Y^{(0)}$$

eliminate repeated control points

$$Y^{(m)} = J^{(m)} Y^{(0)}$$

enhancement matrix

$$\max\{F(t)\} \leq \max\{y_P \mid P \in \mathbb{P}_m\} \longrightarrow \max\{F(t)\} \leq \max\{J^{(m)} F^B\}$$

➢ **Inequality equation** $F(t) \leq c \Leftarrow J^{(m)} F^B \leq c$

Operation Matrix of BP & Transformation – Part 3

Basic idea: Use $(n-1)$th-order polynomial to approximate nth-order terms

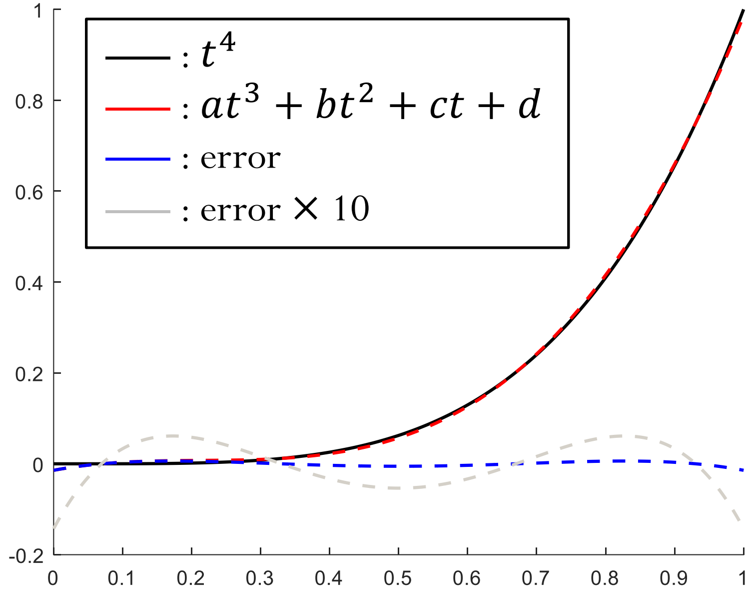
$$t^4 \approx at^3 + bt^2 + ct + d$$

$$\int_0^t B_3(t)\,\mathrm{d}t \approx \boldsymbol{L}B_3(t)$$

operational matrix

$$\frac{\mathrm{d}\boldsymbol{P}(\boldsymbol{t})}{\mathrm{d}\boldsymbol{t}} = \boldsymbol{F}(\boldsymbol{t}) \Rightarrow \int_0^t \frac{\mathrm{d}\boldsymbol{P}(\boldsymbol{t})}{\mathrm{d}\boldsymbol{t}}\mathrm{d}\boldsymbol{t} = \int_0^t \boldsymbol{F}(\boldsymbol{t})\mathrm{d}\boldsymbol{t} \Rightarrow$$

$$\boldsymbol{P}(\boldsymbol{t}) - \boldsymbol{P}(\boldsymbol{0}) = \int_0^t \boldsymbol{F}(\boldsymbol{t})\mathrm{d}\boldsymbol{t} = \left(\boldsymbol{F}^{\boldsymbol{B}}\right)^{\boldsymbol{T}} \int_0^t \boldsymbol{B}_3(\boldsymbol{t})\,\mathrm{d}\boldsymbol{t} \approx \left(\boldsymbol{F}^{\boldsymbol{B}}\right)^{\boldsymbol{T}} \boldsymbol{L}\boldsymbol{B}_3(\boldsymbol{t})$$



Maximum error: 0.006122

Average error: 0.004104

Ref: S. A. Yousefi, M. Behroozifar. Operational matrices of Bernstein polynomials and their applications. International Journal of Systems Science. 2010, 41(6): 709-716

Transformation Rules

According to $F(t) = (F^B)^\mathrm{T} B_3(t)$, we have

- Integral term from 0 to 1 $$\int_0^1 F(t)\mathrm{d}t = 1^\mathrm{T} F^B/4$$
- Integral term from 0 to t $$\int_0^t F(t)\mathrm{d}t \approx (F^B)^\mathrm{T} L B_3(t)$$
- Derivative term $$\frac{\mathrm{d}F(t)}{\mathrm{d}t} = (W_3 F^B)^T B_2(t)$$
- Equality equation $$F(\tau) = 0 \Leftrightarrow F^{B,k} = 0$$
- Inequality equation $$F(t) \le c \Leftarrow J^{(m)} F^B \le c$$

Mixed-integer linear reformulation

Objective: Minimize operation cost

$$\min \sum_{\tau}\sum_{i}\Big(\underbrace{c_{su,i}U_{g,i,\tau} + c_{sd,i}D_{g,i,\tau}}_{\text{startup \& shutdown cost}} + \underbrace{c_g^{PR}R_{g,i,\tau}^{PR} + c_w^{PR}R_{w,i,\tau}^{PR}}_{\text{PFR reserve cost}}\Big) + \sum_{\tau}\sum_{i}\sum_{s}\underbrace{\omega_s F_{g,i,\tau}^{s}}_{\text{expected fuel cost}}$$

Subject to:

- **Discrete-time constraints** – mixed-integer linear equations, tractably handled by solvers
- **Continuous-time constraints → mixed-integer linear equations**

$$\frac{2H_{sys}}{h_{\tau'}}\left(\Delta f_{\tau'}^{B} - \Delta f_{\tau',ini}^{B}\right) + k_D P_d L^{\mathrm{T}}\Delta f_{\tau'}^{B} = L^{\mathrm{T}}\left(\Delta P_d^{B} - P_{sys,\tau'}^{PR,B}\right)$$

$$\Delta f_{\tau',ini}^{B,k}|_{\tau'=1} = \Delta f_{DB},\ \ \Delta f_{\tau',ini}^{B,k}|_{\tau'>1} = \Delta f_{\tau'-1}^{B,3}$$

$$\frac{T_{g,i}}{h_{\tau'}}\left(P_{g,i,\tau'}^{PR,B} - P_{g,i,\tau',ini}^{PR,B}\right) + L^{\mathrm{T}}P_{g,i,\tau'}^{PR,B} = G_{g,i}I_{g,i}L^{\mathrm{T}}\left(\Delta f_{\tau'}^{B} - \Delta f_{DB}^{B}\right)$$

$$P_{g,i,\tau',ini}^{PR,B,k}|_{\tau'=1} = 0,\ \ P_{g,i,\tau',ini}^{PR,B,k}|_{\tau'>1} = P_{g,i,\tau'-1}^{PR,B,3}$$

$$P_{w,i,\tau'}^{PR,B} = G_{w,i}\left(\Delta f_{\tau'}^{B} - \Delta f_{DB}^{B}\right)$$

$$2\bar{\dot{f}}H_{sys,\tau} \geq \Delta P_{d,\tau} \qquad J\Delta f_{\tau'}^{B} \leq \overline{\Delta f}$$

$$k_D P_d \overline{\Delta f_{err}} + G_{sys,\tau}\left(\overline{\Delta f_{err}} - \Delta f_{DB}\right) \geq \Delta P_{d,\tau}$$

$$JP_{g,i,\tau,\tau'}^{PR,B} \leq R_{g,i,\tau}^{PR} \qquad JP_{w,i,\tau,\tau'}^{PR,B} \leq R_{w,i,\tau}^{PR}$$

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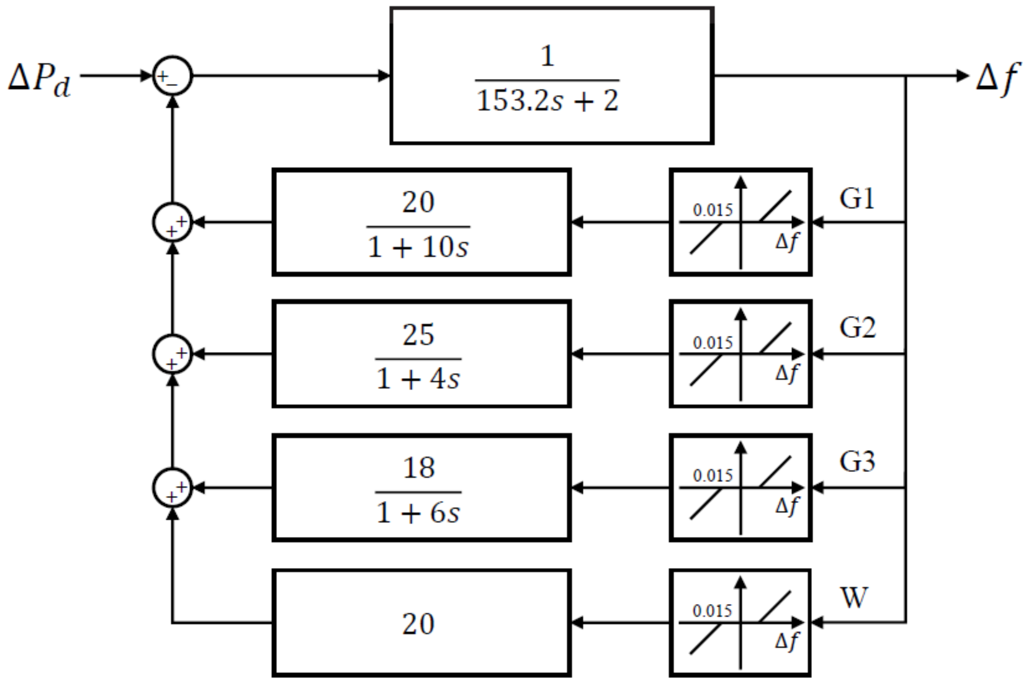
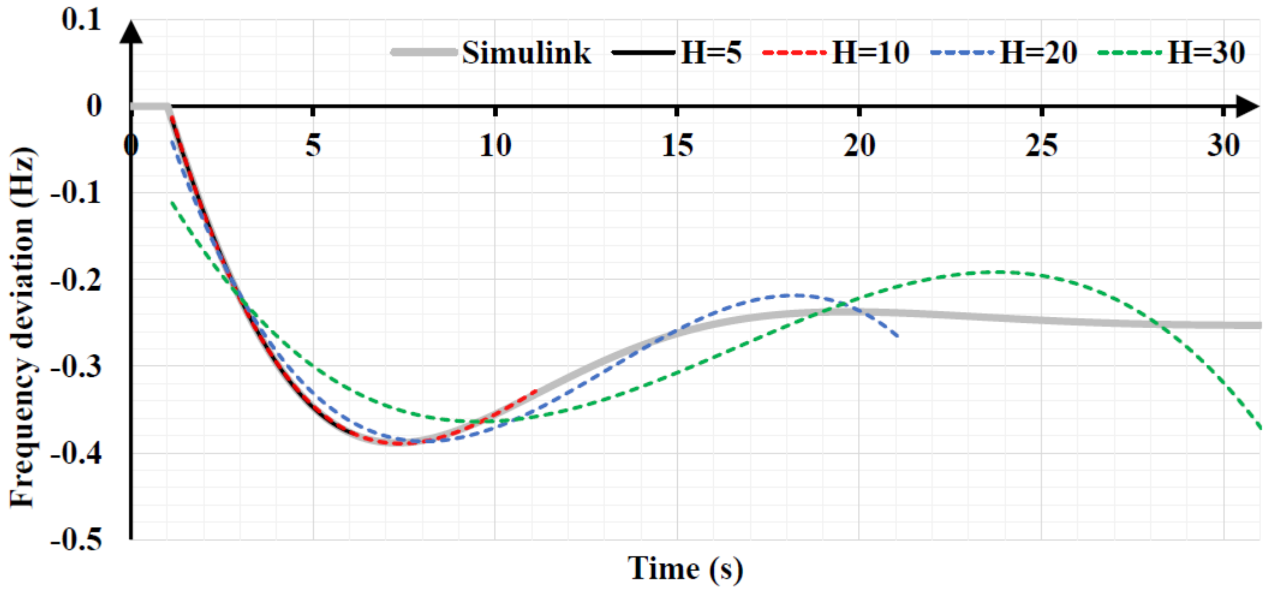
An Example



H is the horizon considered for frequency dynamics



NADIR VALUES AND THEIR RELATIVE ERRORS UNDER DIFFERENT H

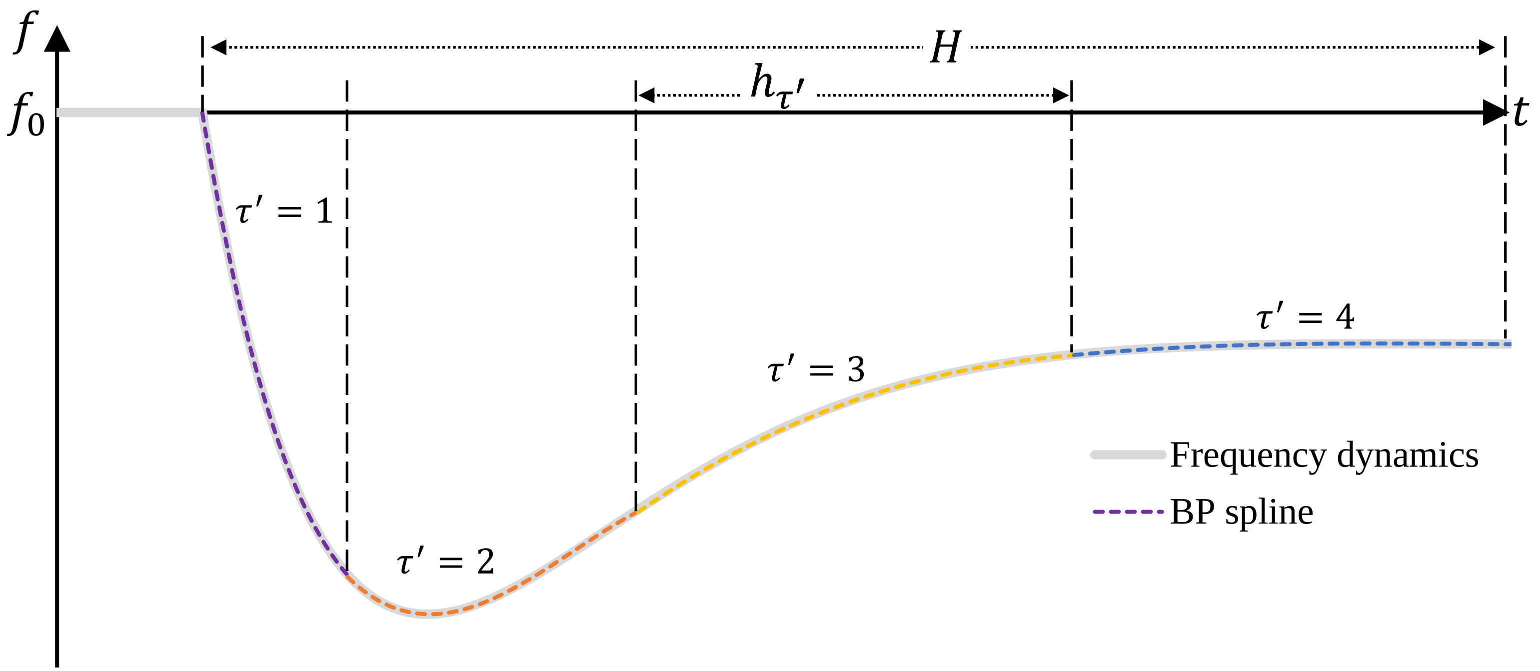
	Simulink	BP approximation			
		H=5s	H=10s	H=20s	H=30s
Nadir value (Hz)	−0.3884	−0.3778	−0.3892	−0.3866	−0.3759
Relative error	N/A	2.73%	0.20%	0.46%	3.21%

- A shorter H results in a higher accuracy, but may not cover the frequency nadir
- A conservative H causes a poor accuracy

Segment-wise Approximation

1-segment BP approximation is not able to fit frequency dynamics for infinitely long

- A multi-segment BP approximation produces better performance
- The number of constraints increases linearly with the number of segments.

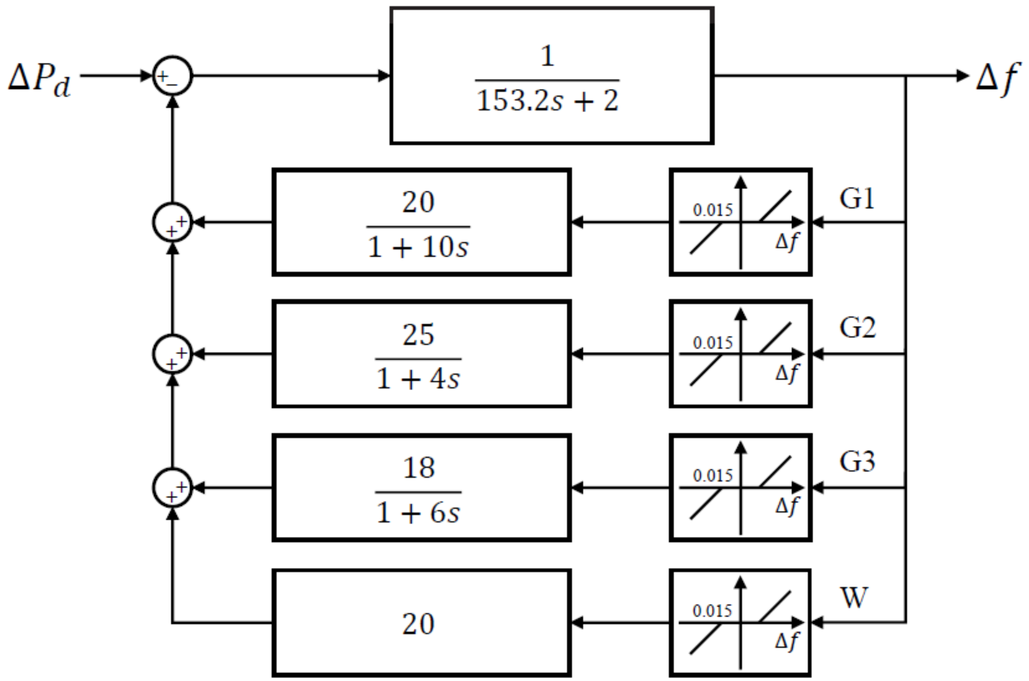


We suggest an **uneven division of H**

➢ the early segments should be shorter for a more accurate estimate of nadir

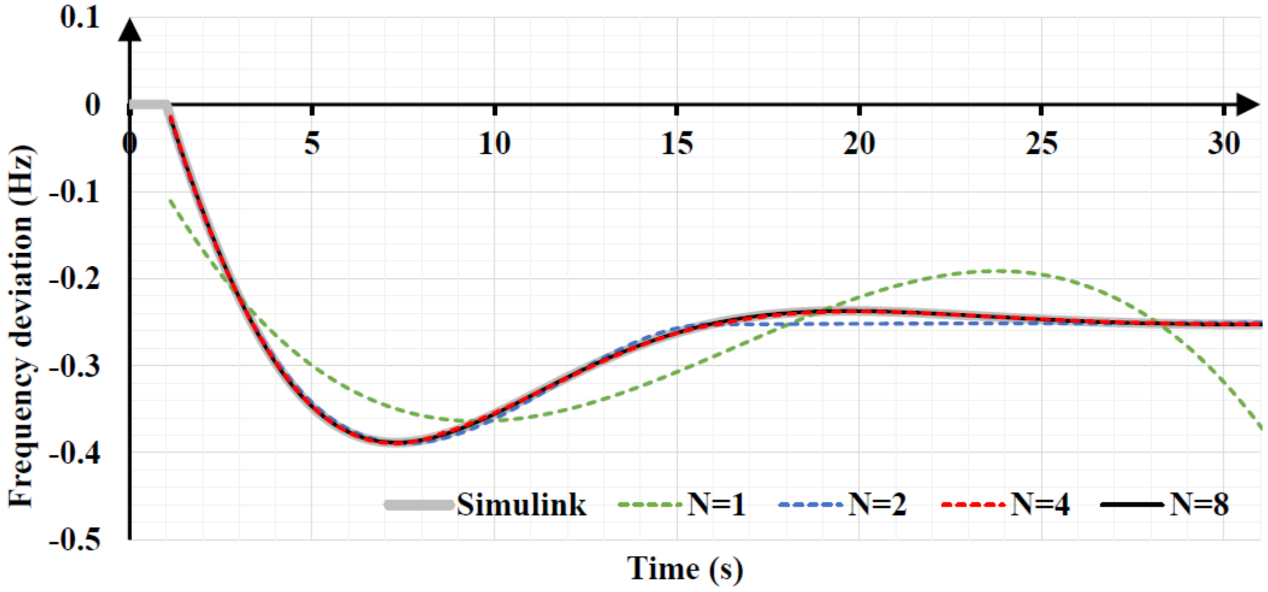
➢ the latter segments should be longer to relieve computational burden

An Example



More segments bring a higher accuracy but increase the number of variables and constraints

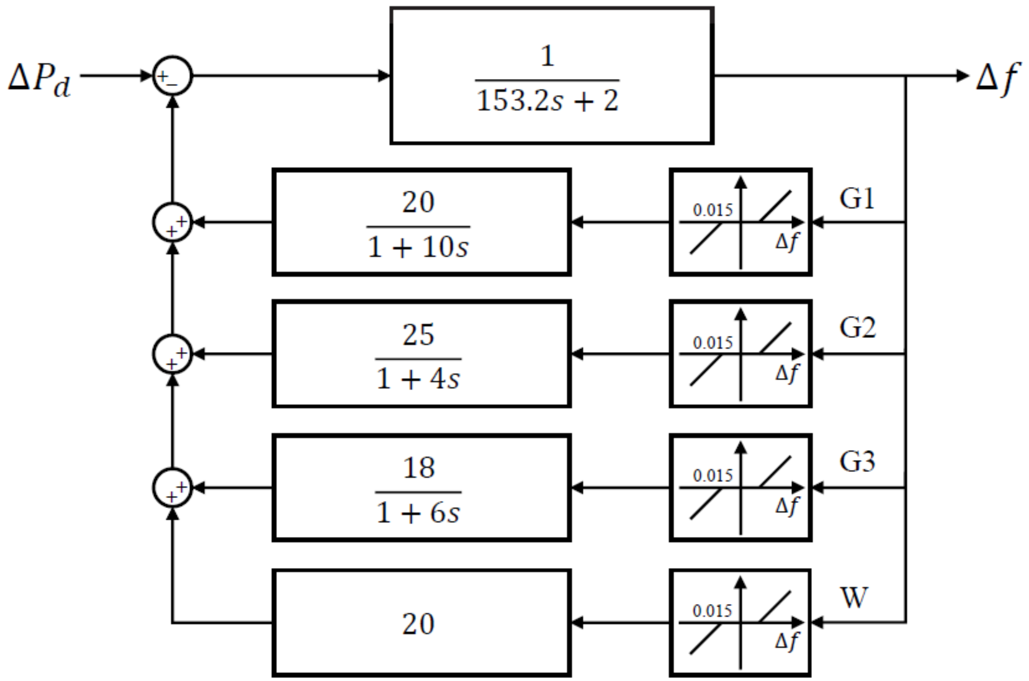
$H = 30\text{s}$, even division



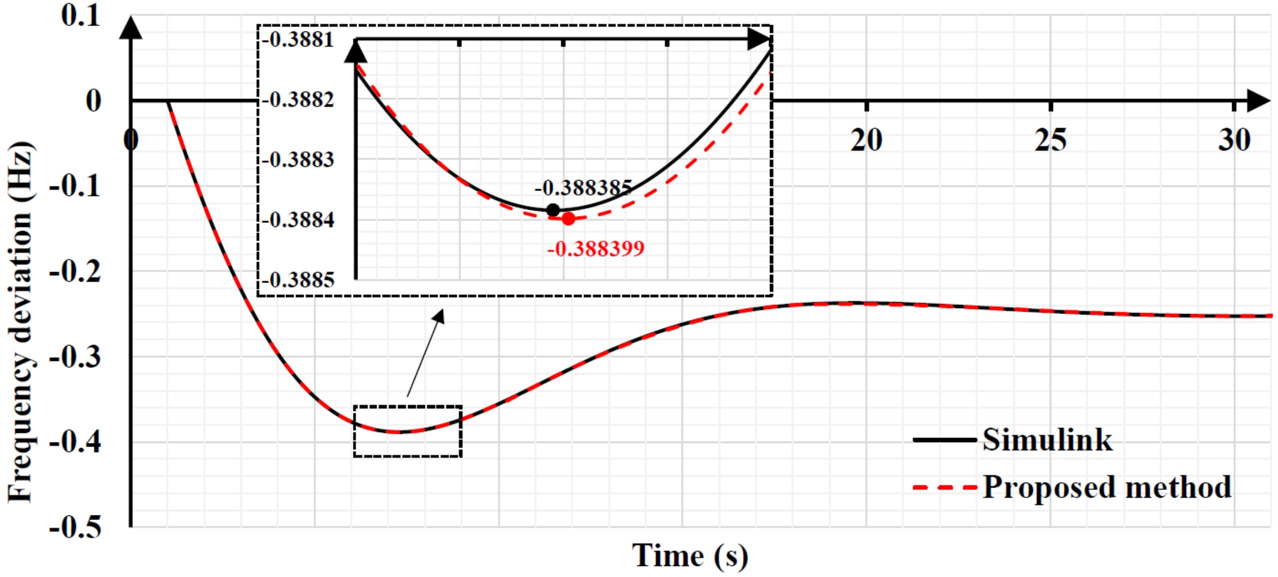
NADIR VALUES AND THEIR RELATIVE ERRORS UNDER DIFFERENT N

	Simulink	BP approximation			
		$N = 1$	$N = 2$	$N = 4$	$N = 8$
Nadir value (Hz)	−0.3884	−0.3759	−0.3893	−0.3892	−0.3883
Relative error	N/A	3.21%	0.23%	0.20%	0.02%

An Example

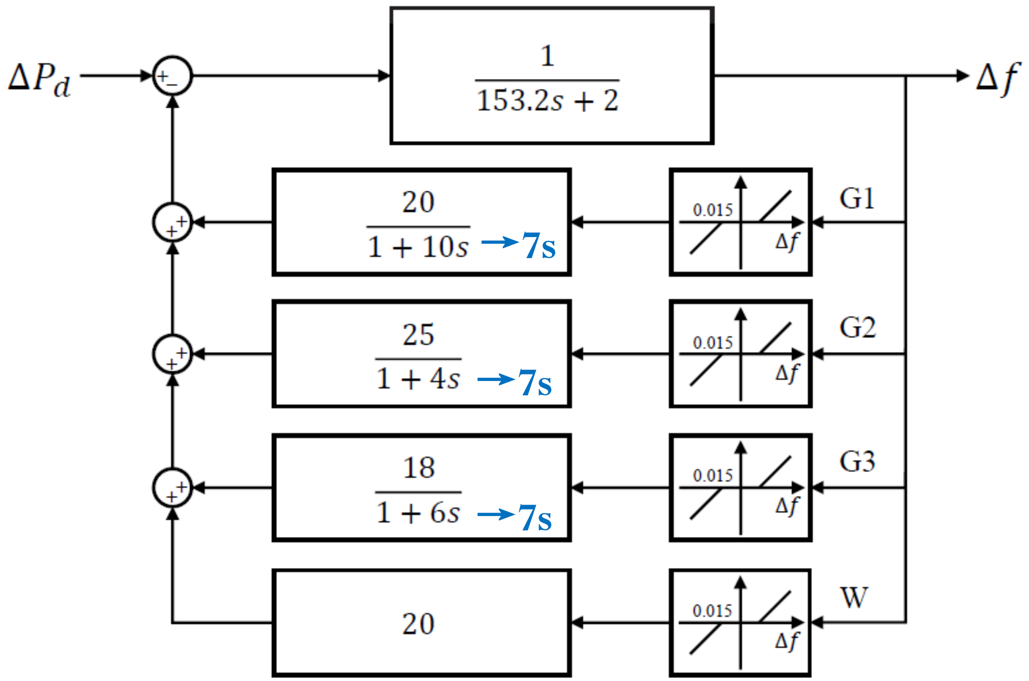


H = 30s, uneven division
(such as 10%, 20%, 30%, 40%)

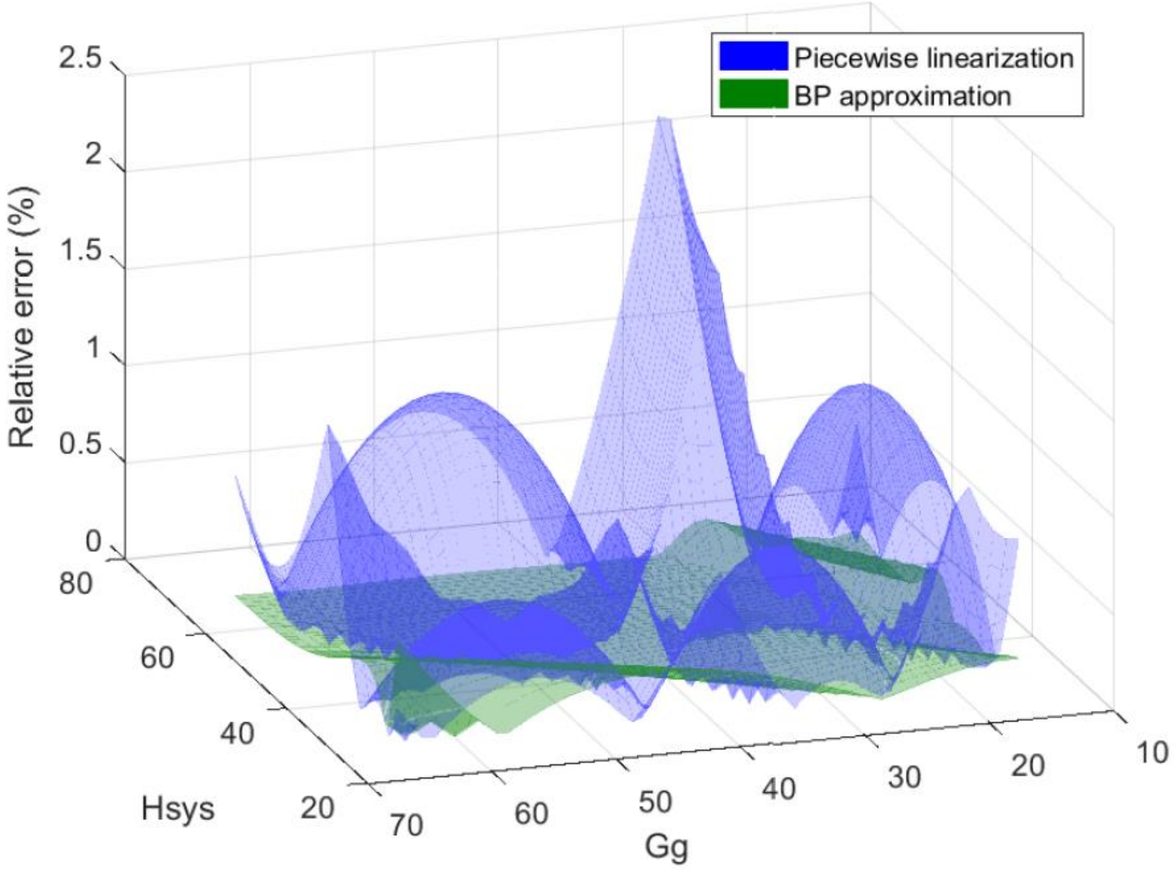


0.004% relative error
good accuracy!

An Example



Compare with the existing piecewise linearization method using 99 evaluation points



The average relative error is one order of magnitude lower than that of piecewise linearization

	Piecewise Linearization	BP approximation
Average relative error	0.5031%	0.0653%

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Conclusion

- We incorporated the frequency dynamics using DAEs into the stochastic UC model and validated the effectiveness in deciding UC and PFR reserves for frequency security
- We adopted BP splines to obtain a linear approximation of the DAEs and demonstrated the high accuracy in depicting frequency dynamics
- The method can consider various control processes, such as the dead band[1]
- The method can be tractably applied to other types of dynamics, such as natural gas dynamics[2], temperature dynamics, etc.

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[1] Bo Zhou, Ruiwei Jiang, Siqian Shen, "Frequency-Secured Unit Commitment: Tight Approximation using Bernstein Polynomials," IEEE Transactions on Power Systems, 2nd review. (arXiv: 2212.12088)
[2] Bo Zhou, et al, "Function-space optimization to coordinate multi-energy storage across the integrated electricity and natural gas system," International Journal of Electrical Power & Energy System, 2023.

Thank You for Attention!